Implementation of adiabatic Abelian geometric gates with superconducting phase qubits

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Abstract. We have developed an adiabatic Abelian geometric quantum computation strategy based on the non-degenerate energy eigenstates in (but not limited to) superconducting phase-qubit systems. The fidelity of the designed quantum gate was evaluated in the presence of simulated thermal fluctuation in superconducting phase qubits circuit and was found to be rather robust against the random errors. In addition, it was elucidated that the Berry phase in the designed adiabatic evolution may be detected directly via the quantum state tomography developed for superconducting qubits.

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Contents

1. Introduction

Quantum computation (QC) has been attracting more and more interests for the past decade due to its unrivaled power exceeds that of the classical counterpart in solving certain problems. Significant and exciting progress has been achieved both theoretically and experimentally in this field. Nevertheless, there are still many difficulties and challenges in physical implementation of quantum computation. To increase the fidelity of quantum gates to an acceptable high level is one of them, and is essential to construct workable quantum logical gates in scalable quantum computers. Recently, several promising schemes based on the geometric phase have been proposed for achieving built-in fault-tolerant quantum gates with high fidelities[1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11], as it is believed that QC based on geometric phase shifts may be more robust against certain type of stochastic errors in the control/operation parameters/processes than dynamic quantum gates. For the past several years, based on the adiabatic non-Abelian geometric phase shifts accumulated for degenerate/dark energy eigenstates, the holonomic quantum computation (HQC) has been proposed and developed [5, 6, 7]. In addition, a geometric QC scheme based on the nonadiabatic but cyclic geometric phases has been achieved [8]. On the other hand, the environment effects on the Berry phase of a two-level system have also been addressed[12].

On the other hand, it has been realized that superconducting qubits provide us a promising approach towards a scalable solid-state quantum computer [13, 14, 15, 16, 17]. However, the unavoidable random errors may lead to a serious reduction of the fidelities of the wanted quantum gates. Constructing fault-tolerant quantum logic gates in superconducting based on geometric phase has been paid particular attention recently [2, 3, 7, 8]. Being different from the above schemes, we here develop an adiabatic Abelian geometric QC scheme based on the non-degenerate energy eigenstates (considering

qubit-qubit interaction as $\sigma_y\sigma_y$ type), and then analyze its fidelity against certain kind of simulated noises in superconducting phase qubits.

The paper is organized as follows. In section 2, we introduce the adiabatic Abelian geometric gates. In section 3, we simulate the fidelities of both the single-qubit and the two-qubit controlled adiabatic Abelian geometric gate in the presence of random fluctuations. In section 4, the detection of the Berry phase in superconducting phase qubits is analyzed. Section 5 presents relevant discussions and a brief summary.

2. Adiabatic Abelian Geometric Gates

We now elaborate our adiabatic Abelian geometric QC strategy. A qubit system, when the system Hamiltonian with two instantaneous non-degenerate energy levels changes adiabatically and cyclically in a parameter space with the period τ, behaves like a spin$\frac{1}{2}$ particle in a magnetic field, with the Hamiltonian as $H = \vec{\sigma}\cdot\mathbf{B}/2$. Under the adiabatic approximation, the two orthogonal energy eigenstates $|\psi_\pm(t)\rangle$ will also follow the Hamiltonian to evolve adiabatically and cyclically starting from the initial states $|\psi_\pm(0)\rangle$: $|\psi_\pm(\tau)\rangle = U(\tau)|\psi_\pm(0)\rangle \approx \exp(\pm i\gamma)|\psi_\pm(0)\rangle$, where the $U(\tau)$ is the evolution operator of the system and the $\pm\gamma$ are respectively the total phases accumulated for the $|\psi_\pm\rangle$ states in the evolution. If we denote $|\psi_+(0)\rangle = \cos\frac{\xi}{2}|0\rangle + e^{i\eta}\sin\frac{\xi}{2}|1\rangle$ and $|\psi_-(0)\rangle = -\sin\frac{\xi}{2}|0\rangle + e^{i\eta}\cos\frac{\xi}{2}|1\rangle$, with $|0\rangle$ and $|1\rangle$ as the two eigenstates of σ_z and being chosen as our computational basis ($\eta = 0$ for $B_y(0) = 0$ and $\eta = \pi/2$ for $B_x(0) = 0$).

Thus, for an arbitrary initial state of the system $|\psi_{in}\rangle = a_+|\psi_+(0)\rangle + a_-|\psi_-(0)\rangle$ with $a_\pm = \langle\psi_\pm(0)|\psi_{in}\rangle$, after the adiabatic and cyclic evolution time τ, the final state is found to be $|\psi_f\rangle \approx U(\gamma,\xi,\eta)|\psi_{in}\rangle$, where

$$U = \begin{pmatrix} e^{i\gamma}\cos^2\frac{\xi}{2} + e^{-i\gamma}\sin^2\frac{\xi}{2} & ie^{-i\eta}\sin\xi\sin\gamma \\ ie^{i\eta}\sin\xi\sin\gamma & e^{i\gamma}\sin^2\frac{\xi}{2} + e^{-i\gamma}\cos^2\frac{\xi}{2} \end{pmatrix}. \qquad (1)$$

Moreover, a controlled two-qubit gate may also be achieved under the condition that the control qubit is off resonance in the operation of the target qubit (to be addressed later).

Considering that γ is the total phase usually consisting of both geometric and dynamic phases, we here illustrate how to eliminate the corresponding dynamic phase in a simple two-loop quantum gate operation, so that the achieved U-gate is a pure geometric one depending only on the geometric phase accumulated in the whole evolution. Set the basic adiabatically cyclic evolution time to be τ_0 with the corresponding geometric Berry phase as γ_g^0. After the first cyclic evolution of the states $|\psi_\pm\rangle$ by driving the fictitious field adiabatically with the period τ_0, we reverse promptly the fictitious field direction such that the states $|\psi_\pm\rangle$ are unchanged, i.e., $\mathbf{B}(\tau_0+0) = -\mathbf{B}(\tau_0)$ and $|\psi_\pm(\tau_0+0)\rangle = |\psi_\pm(\tau_0)\rangle$. Then we let $\mathbf{B}(\tau_0+t) = -\mathbf{B}(t)$ in the second τ_0-time cycle evolution. During the second period, the state $|\psi_+\rangle$ ($|\psi_-\rangle$) acquires the same geometric phase as that in the first period but with the reversal sign of the dynamic phase, so that the accumulated total phase of $|\psi_+\rangle$ ($|\psi_-\rangle$) at the end

second period is a pure geometric phase $\gamma = 2\gamma_g^0$ $(-2\gamma_g^0)$. Therefore, the pure geometric quantum $U(2\tau_0)$-gates given by equation (3) can be obtained. For example, two simple noncommutable single-qubit gates, the type of Hadamard-gate and the type of NOT gate, can be achieved by setting ($\xi = \pi/4$, $\eta = 0$, $\gamma_g^0 = \pi/4$) and ($\xi = \pi/2$, $\eta = 0$, $\gamma_g^0 = \pi/4$), respectively.

3. Fidelity of Adiabatic Abelian gates

Recently, it was reported from numerical simulations that the earlier proposed two kinds of geometric quantum gates, a class of non-Abenian holonomic gates [5] and a set of nonadiabatic Abelian geometric gates [8, 9], are likely more robust against stochastic control errors than dynamical gates [10, 11]. It is natural to ask whether the present adiabatic Abelian geometric gates are also robust against stochastic errors as expected. To answer this question, here we illustrate this by superconducting phase qubits.

As is known, a large current-biased Josephson junction(figure 1a, c) may work as a typical phase qubit, which can be considered as an anharmonic LC resonator with resonance frequency $\omega_p = (L_J C_J)^{-1/2}$, whose two lowest quantized engergy levels are chosen as the qubit states[16, 17, 18], where L_J is the Josephson inductance and C_J is the junction capacitance. The Josephson inductance is given by $L_J = \phi_0/2\pi I_c \cos\delta$, where I_c is the junction critical current, δ is the phase difference across the junction given through $I = I_c \sin\delta$, and $\phi_0 = h/2e$ is the superconducting flux quantum. As the junction bias current I close to the critical current I_c, the anharmonic potential may be approximated by a cubic potential parameterized by the potential barrier height $\Delta U(I) = (2\sqrt{2} I_c \phi_0/3\pi)[1 - I/I_c]^{3/2}$ and a plasma oscillation frequency at the bottom of the well $\omega_p(I) = 2^{1/4}(2\pi I_c/\phi_0 C)^{1/2}[1 - I/I_0]^{1/4}$. Microwave induces transitions between levels at a frequency $\omega_{mn} = E_{mn}/\hbar = (E_m - E_n)/\hbar$, where E_n is the energy of state $|n\rangle$. The state of the qubit can be controlled with dc and microwave pulses of bias current $I(t) = I_{dc} + \delta I_{dc}(t) + I_{\mu w}(t)\cos\phi\cos\omega_{10}t + I_{\mu w}(t)\sin\phi\sin\omega_{10}t$. As usual, under a reasonable approximation that the dynamics of the system is restricted to the Hilbert space spanned by the lowest two states, the Hamiltonian in the ω_{10} rotating frame may be written as

$$H = \hat{\sigma}_x I_{\mu w}(t)\cos\phi\sqrt{\hbar/2\omega_{10}C}/2 + \hat{\sigma}_y I_{\mu w}(t)\sin\phi\sqrt{\hbar/2\omega_{10}C}/2 \\ + \hat{\sigma}_z \delta I_{dc}(t)(\partial E_{10}/\partial I_{dc})/2, \tag{2}$$

where $\hat{\sigma}_{x,y,z}$ are Pauli operators. As schematically shown in figure 1b, a untrivial two-qubit gate could be constructed by capacitive coupling.

From equation (2), one could define a fictitious field $\mathbf{B} \equiv (\nu\cos\phi, \nu\sin\phi, \Delta\omega)$, where $\nu = I_{\mu w}(t)\sqrt{\hbar/2\omega_{10}C}, \Delta\omega = \delta I_{dc}(t)(\partial E_{10}/\partial I_{dc})$. The phase qubit thus behaves like a spin-$\frac{1}{2}$ particle in a magnetic field, with the Hamiltonian as $H = \vec{\sigma}\cdot\mathbf{B}/2$. For such a quantum system, the acquired geometric phase of its energy eigenstate is equal to half of the solid angle subtended by the area in the parameter space enclosed by the closed evolution loop of the fictitious magnetic field. The solid angle may be evaluated

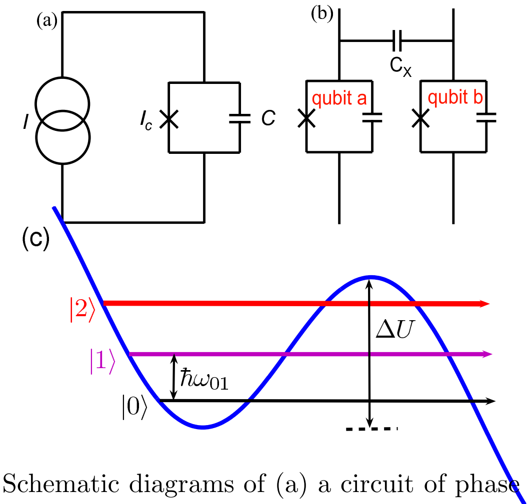

Figure 1. Schematic diagrams of (a) a circuit of phase qubit; (b) a two-qubit gate, where two single phase qubits are coupled by a capacitor C_x; (c) quantized energy levels in a current biased Josephson Junction, where the two lowest eigenstates $|0\rangle$ and $|1\rangle$ form a qubit.

by [8]

$$\Omega = \int_0^\tau \frac{B_x \partial_t B_y - B_y \partial_t B_x}{|B|(B_z + |B|)} dt, \tag{3}$$

under the condition $\mathbf{B}(\tau) = \mathbf{B}(0)$. Especially, when the adiabatic evolution path forms a cone in the parameter space $\{\mathbf{B}\}$ under the varying current, the corresponding Berry phases of two energy eigenstates are simply given by [1] $\gamma_g = \pm\pi[1 - \Delta\omega/\sqrt{(\Delta\omega)^2 + (\nu)^2}]$.

We will perform ceratin kind of numerical simulations on the fidelity of the adiabatic Berry phase gates given by equation (1) and (8), subject to the modelled random noises for the weakly fluctuated driving bias current. Note that, in the numerical studies of Refs.[10, 11], the fluctuations of control parameters were assumed to be uniformly distributed in an interval and merely a certain type of states in the Bloch sphere were sampled to evaluate the average fidelity of gates. While in real experiments, the finite impedance of the bias-current source produces the decoherence of superconducting phase qubit from the dissipation and noises. We here mainly consider the noise in the current due to the thermal fluctuation, which is likely one of the main noise sources in superconducting qubits circuit[19]. The actual noise current generated by a resistance R at temperature T may be estimated by $I_n(rms) = (4k_B TB/R)^{1/2}$, where B is the bandwidth parameter [20]. The amplitude of the noise current would obey a Gaussian distribution[17]. In fact, assuming the critical current of superconducting phase qubits $I_c \sim 10\mu$A[21], the measurement bandwidth $B \sim 10$GHz and the bias resistor R$\sim$10KΩ at T$\sim$4.2K, the total current noise would be around 15 nA. The bias current is driven close to the critical current I_c and the transition frequency between qubit states is $\omega_{10}/2\pi \sim 6$GHz. The Rabi frequency is $\nu/2\pi \sim$300MHz and the Ramsey frequency is $\Delta\omega/2\pi \sim 300$MHz. The fluctuation of Ramsey frequency resulting from noise in the bias current is about 10MHz. We will below evaluate the average fidelity of the designed

new geometric quantum gates subject to this type of errors for any input state.

As is known, the average fidelity of a quantum logic gate in the presence of random noises may be defined as

$$\overline{F} = \lim_{N\to\infty} \frac{1}{N} \sum_{j=1}^{N} |\langle\psi_{in}|\widehat{U}^{\dagger}\widehat{U}^{j}_{noise}|\psi_{in}\rangle|^2, \tag{4}$$

where $|\psi_{in}\rangle = [\cos(\theta_i/2), e^{i\varphi_i}\sin(\theta_i/2)]^T$($T$ represents the transposition of matrix.), $\theta_i \in [0,\pi]$ and $\varphi_i \in [0,2\pi]$ are the coordinators of the input state in our numerical simulations. Here, U is an ideal adiabatic quantum gate denoted by equation (3) in the absence of random errors and U_{noise} is the gate operator in the presence of random errors.

For simplicity, we here focus only on a cone-type adiabatic evolution: $\xi = \tan^{-1}(\nu/\Delta\omega)$. Since this type of adiabatic evolution of the field could have various forms, it seems subtle to directly simulate the state evolution with a reliable way under the adiabatic condition in the presence of random errors. To evade this subtle issue, we adopt a simple method to model effectively the effect of random errors occurred in the evolution. For a given configuration of errors in the evolution, let us look at the final state $|\tilde{\psi}_+\rangle = U_{noise}|\psi_+\rangle(0)$ and regard it to be evolved adiabatically and cyclically as well as ideally from a visual initial state $|\tilde{\psi}_+\rangle(0) = \cos\frac{\tilde{\xi}}{2}|0\rangle + e^{i\tilde{\eta}}\sin\frac{\tilde{\xi}}{2}|1\rangle = U^{-1}(\tilde{\gamma},\tilde{\xi},\tilde{\eta})|\tilde{\psi}_+\rangle$, namely, $|\tilde{\psi}_+\rangle = e^{i\tilde{\gamma}}|\tilde{\psi}_+\rangle(0)$. In this sense, U_{noise} can be expressed as $U(\tilde{\gamma},\tilde{\xi},\tilde{\eta})$ in equation (3) with $\tilde{\gamma}$ as the geometric Berry phase of the two loops. Here, the random parameters $(\tilde{\gamma},\tilde{\xi})$ may be determined by the randomly fluctuated bias current from the relations γ^0_g $(\nu,\, \Delta\omega)$ and ξ $(\nu,\, \Delta\omega)$, with the Gaussian-type error probability density

$$\frac{dp(x)}{dx} = \frac{\exp(-x^2/2\sigma^2)}{\sqrt{2\pi}\sigma}, \tag{5}$$

where the x is the deviation from ν (or $\Delta\omega$, η), and σ is the mean squared noise.

3.1. Fidelity of single-qubit gates

In the numerical simulations reported here, we randomly choose more than ten thousand stochastic numbers ($N \geq 10000$) for a given mean squared noise σ (for brevity but without loss of generality, we hereafter set $\tilde{\eta} = \eta = 0$ and neglect its randomness). We select the experimental parameter of superconducting phase qubits: $\omega_{10}/2\pi = 6$GHz and $\sigma_0 = \sigma_1 = 0.1$, where σ_0 and σ_1 represents the fluctuation of $\Delta\omega$ and ν respectively. We then calculate the average fidelity (up to satisfactory convergence) versus the coordinates of the initial state and the parameters, as depicted in figures 2, 3 and 4, respectively. Several remarkable features can be seen from the figures. (i) The calculated fidelity of geometric quantum gates for any input state is rather high (larger than 0.92) for the considered noises. Actually, the amplitude of microwave current could be controlled precisely in experiments. Therefore, the σ_1 is much smaller than σ_0 in superconducting phase qubits. From the above disscusions, if the σ_0 is about 0.03, the designed geometric quantum gate is likely rather insensitive to the stochastic errors. (ii)The suppression effect of $\Delta\omega$ fluctuations on the fidelity is weaker than that of ν fluctuations, which

Figure 2. The fidelity of single-qubit gate in the presence of $\Delta\omega$-fluctuations, where $\varphi_i = 0$ in (a) and (b), $\theta_i = \pi/2$ in (c) and (d). Parameters are: $\omega_{10}/2\pi = 6$GHz and $\sigma_0 = 0.1$.

Figure 3. The fidelity of single-qubit gate in the presence of ν-fluctuations, where $\varphi_i = 0$ in (a) and (b), $\theta_i = \pi/2$ in (c) and (d). Parameters are: $\omega_{10}/2\pi = 6\text{GHz}$ and $\sigma_1 = 0.1$.

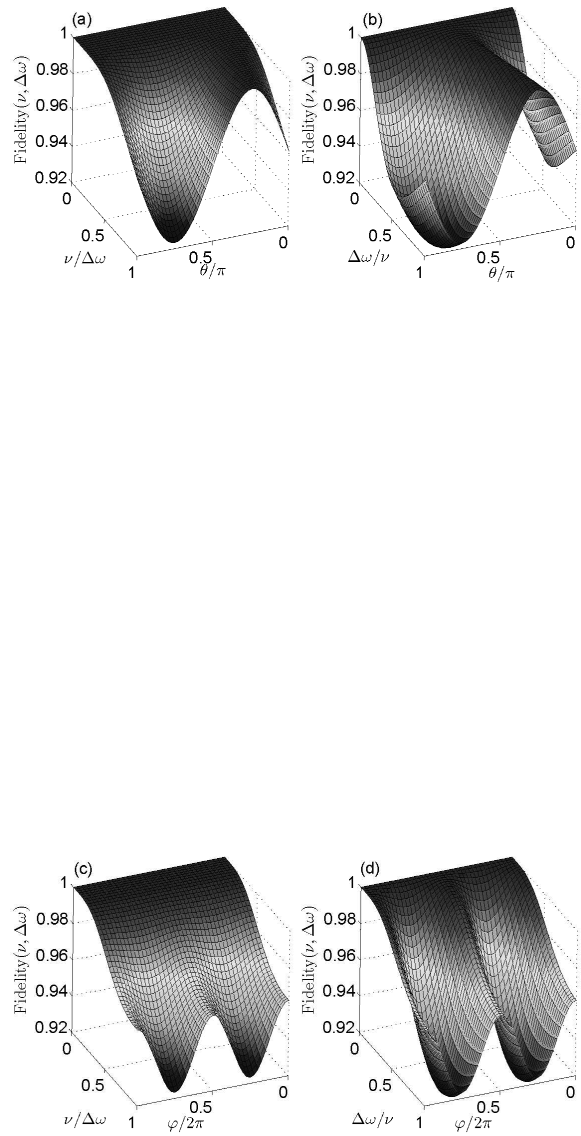

Figure 4. The fidelity of single-qubit gate in the presence of fluctuations on both $\Delta\omega$ and ν, where $\varphi_i = 0$ in (a) and (b), $\theta_i = \pi/2$ in (c) and (d). Parameters are: $\omega_{10}/2\pi = 6\text{GHz}$, $\sigma_0 = 0.1$, and $\sigma_1 = 0.1$.

becomes more pronounced when the mean squared noise is stronger(not shown here). Also reasonablly, the joint effect of both $\Delta\omega$ and ν fluctuations on the fidelity is relatively stronger than any single one, but the shapes of the cooresponding figures are similar. Note that, since it seems difficult to control the $\Delta\omega$ precisely because $\Delta\omega$ often varies due to the noise current in experiments [22], to optimize a quantum gate with the persent geometric scenario may be quite helpful. (iii) The fidelity is very close to 1 for $\nu \ll \Delta\omega$ or $\nu \gg \Delta\omega$. Actually, we have a trivial geometric phase 2π and a trivial unit gate in this case. (iv) For a given ξ, the fidelity reaches a maximum when the input state is the eigenstate of the Hamiltonian.

3.2. Fidelity of two-qubit gates

We now turn to address a kind of non-trivial two-qubit controlled phase gate in the present system. So far, many efforts have been paid to a two-qubit controlled phase gate with $\sigma_z\sigma_z$ coupling[2, 3, 4, 8]. However, the present two superconducting phase qubits are coupled with a capacitor [see figure 1(b)], and thus the qubit-qubit interaction takes the $\sigma_y\sigma_y$ form, with the total Hamiltonian being given by

$$\hat{H} = \textstyle\sum_{i=a,b}\hat{H}_i + \frac{J}{2}\hat{\sigma}_y^{(a)}\hat{\sigma}_y^{(b)}, \tag{6}$$

where the coupling strength $J \approx (C_x/C_J)\hbar\omega_{01}$. This Hamiltonian could be used to manipulate the target qubit (*qubit b*) for the realization of a two-qubit gate under the condition that the control qubit (*qubit a*) is off resonance in the operation of the target qubit. The Hamiltonian of *qubit b* in the ω_{10}^b rotating frame is dependent on the state of *qubit b* through the coupling term J: the contribution is $J/2$ (or $-J/2$) if the state of *qubit a* is $|\psi_a\rangle = |-\rangle$ (or $|\psi_a\rangle = |+\rangle$), with $|-\rangle$ and $|+\rangle$ as the two eigenstates of σ_y. Setting $\tilde{\eta} = \eta = \pi/2$ and after an adiabatic evolution loop, the acquired geometric phase of target qubit is derived as

$$\begin{aligned}\gamma_g^+ &= \frac{1}{2}\int_{\frac{\pi}{2}}^{\frac{5\pi}{2}} \frac{\nu^2 - \frac{1}{2}a\sin\phi}{(\sqrt{-a\sin\phi + b})(\sqrt{-a\sin\phi + b} + \Delta\omega)}d\phi,\\ \gamma_g^- &= \frac{1}{2}\int_{\frac{\pi}{2}}^{\frac{5\pi}{2}} \frac{\nu^2 + \frac{1}{2}a\sin\phi}{(\sqrt{a\sin\phi + b})(\sqrt{a\sin\phi + b} + \Delta\omega)}d\phi,\end{aligned} \tag{7}$$

where $a = \nu J, b = \nu^2 + \Delta\omega^2 + \frac{1}{4}J^2$. Although $\gamma_g^+ = \gamma_g^- = \gamma/2$, we can still have a nontrivial two-qubit controlled geometric phase gate, given by

$$U_{ctrl} = \begin{pmatrix} U_{(\gamma,\xi^+)} & 0 \\ 0 & U_{(\gamma,\xi^-)} \end{pmatrix}, \tag{8}$$

where $\xi^\pm = \tan^{-1}[(\nu \mp J/2)/\Delta\omega]$.

We now numerically simulate the fidelity of the two-qubit gate by assuming only $\Delta\omega$ fluctuations, which are believed to be more significant than ν fluctuations in superconducting phase qubits. The input state is $(\cos\frac{\theta_i}{2}|+\rangle + \sin\frac{\theta_i}{2}|-\rangle)_C \otimes |0\rangle_T$, with subscripts C and T mean the state of the control and target qubits, respectively.

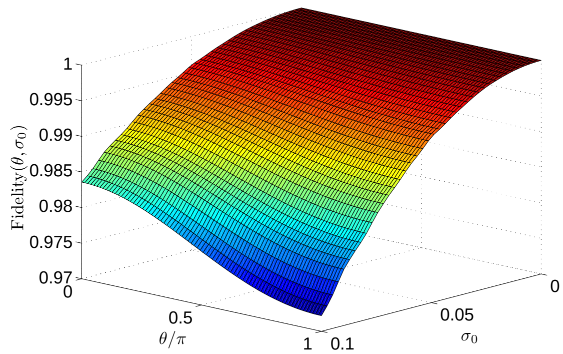

Figure 5. The fidelity of the two-qubit gate in the presence of the $\Delta\omega$ fluctuation with various σ_0 when the input state is $(\cos\frac{\theta_i}{2}|+\rangle + \sin\frac{\theta_i}{2}|-\rangle)_C \otimes |0\rangle_T$. Parameters are: $\omega_{10}^a/2\pi \neq \omega_{10}^b/2\pi = 6$GHz, $\Delta\omega/2\pi = \nu/2\pi = 300$MHz, and $J/2\pi = 150$MHz.

Indeed, for the typical experiment parameters: $\omega_{10}/2\pi \sim 6$GHz, $\Delta\omega/2\pi = \nu/2\pi = 300$MHz,$C_x \sim 33$fF, $C_J \sim 1.3$pF in Ref.[21, 25, 26], we have $J/2\pi \sim 150$MHz. Therefore, the fidelity of the two-qubit gate in the presence of $\Delta\omega$ fluctuation with various σ_0 is shown in figure 5. The fidelities of the two-qubit controlled phase gate are larger than 0.972. It seems that the fidelity of two-qubit gate is also robust against the random errors resulting from thermal fluctuation in superconducting phase qubits as well.

4. Detection of Berry Phase in Superconducting Phase Qubits

Supercondcuting qubits, considered as artificial macroscopic two-level atoms, are also proposed as a candidate for detecting the geometric phase in macroscopic quantum systems[2, 3]. However, these existing proposals suggest to detect the Berry phase through the interference measurement, in which the dephasing may affect seriously the visibility in measuring this phase. In particular, it seems quite difficult to detect the Berry phase via the interference measurement in superconducting phase qubits. Recently, Steffen *et al.*[21] reported the first demonstration of quantum state tomography using single shot measurements in superconducting phase qubits. Stimulated by this experiment, we here propose to directly detect the adiabatic Berry phase and to measure the fidelity of the designed geometric quantum gate via the quantum state tomography in future experiments.

Let us illustrate an example below. The initial state of qubit is prepared as $|\psi_i\rangle = [\cos(\theta/2), \sin(\theta/2)]^T$ in the basis of $[|\psi_+\rangle, |\psi_-\rangle]$ (i.e., $[|0\rangle, |1\rangle]$ if we set $\nu(0) = 0$). As we described before, we drive the field to loop twice in a designated way in the parameter space, and thus the final state is $|\psi_f\rangle = [e^{2i\gamma_g^0}\cos(\theta/2), e^{-2i\gamma_g^0}\sin(\theta/2)]^T$. One rotates the Bloch vector of the qubit state in Bloch sphere with microwave current pulses along x, y and z directions and measures the first excitation $|\psi_-\rangle\langle\psi_-|$ for reconstructing the density matrix of the state. The relative phase change between $|\psi_+\rangle$ and $|\psi_-\rangle$ observed through the state tomography is $4\gamma_g^0$. The corresponding qubit state can be

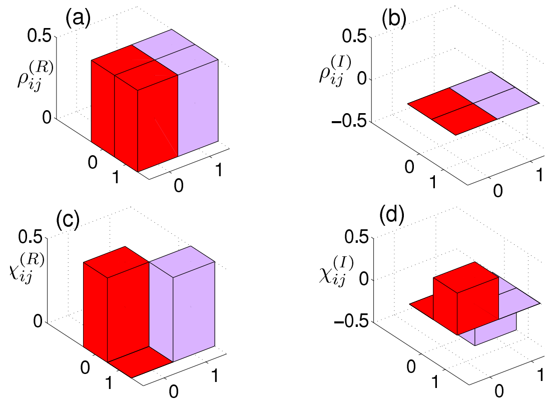

Figure 6. Graphical representation of the density matrices ρ and χ for the initial and final states, with (a)&(c) and (b)&(d) denoting respectively the real and imaginary parts, where $\theta = \pi/2$ and $\gamma_g^0 = \pi/8$.

graphically represented, as shown in figure 6. From this figure, the Berry phase may be determined from the relative phase of the density matrix elements in (or between) the final state (and the initial state). In experiments, one may optimize the experimental result by some methods which help to reduce the unavoidable decoherence and statistical errors [27]. In addition, the target qubit conditional phase shift may be detected by the simultaneous joint measurement of two-qubit state [25, 26].

5. Discussions and Summary

The experimental realization of our proposal for Adiabatic Abelian gates and detecting the Berry phase in superconducting phase qubits is quite possible, although it may meet various technological challenges. The rapid inversion of the bias magnetic field to cancel the the dynamical contribution to the overall phase is experimentally feasible. In fact, with a flux-biased superconducting phase qubit (which is essentially a current-biased Josephson junction) loop size of 50 $(\mu m)^2$[26, 28], changing the flux by about half of a flux quantum in 10^{-10}s, requires sweeping the magnetic field at a rate of about 2×10^5 T/s, that is reachable by current techniques[29].

Perhaps a main challenge is the implementation of the adiabatic evolution of the Hamiltonian to get the Berry phase within the qubits decoherence time, which in turn must be longer than the typical timescale of superconducting phase qubits: $2\pi/\omega_{10}$, $2\pi/(\omega_{10} - \omega_{21})$ ∼3 ns. The slowly varying the phase of microwave current could be realized with 100 linear steps of about 4 ns. The required microwave technique is rather mature [21, 26]. In view of the decoherence time data in Refs.[16] and [21], it seems feasible to detect the geometric phase in phase qubits with the current quantum state tomography technology.

As for a direct comparison of the Adiabatic Abelian gates and the dynamic gates in superconducting qubits, to our knowledge, it was indicated before that the geometric gates are more robust against fluctuations of control parameters than dynamic gates

[11]. From our numerical simulations, adiabatic Abelian geometric gates are likely robust against certain random errors in superconducting phase qubits. We wish to indicate that although there are limitations caused by the adiabatic condition, geometric QC based on the adiabatic Berry phase may have an interesting application in a precise preparation of a quantum state[3, 11], mainly due to its global geometric robustness against certain kind of errors. An experimental process to determine the noisy channel of the controlled qubits based on the qubit state tomography is referred to as quantum process tomography [23, 24]. Since a set of standard qubit states are required to be precisely prepared in the quantum process tomography experiments, we may use the present geometric QC strategy to achieve them. For example, to realize quantum process tomography for a single phase qubit, the four kinds of input states $|0\rangle, |1\rangle, |+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$ and $|-\rangle = (|0\rangle + i|1\rangle)/\sqrt{2}$ need to be precisely prepared. In our scheme, the state $|-\rangle$ can be made from an easy initial state $|0\rangle$ once we set $\varphi_i = 0$, $\theta_i = 0$, $\xi = \pi/2$, and $\gamma_g^0 = \pi/8$, with a relative high fidelity for weaker noises.

In summary, we have developed an adiabatic Abelian geometric QC strategy based on the non-degenerate energy eigenstates. The fidelity of the designed quantum gate has been evaluated in the presence of simulated Gaussian-type thermal fluctuation noises in superconducting phase qubits and found to be rather robust against the random errors. A possible application of our strategic scheme in a precise preparation of designated quantum state has been addressed. We have also proposed to detect directly the Berry phase in phase qubits via the quantum state tomography.

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Note added.—After completion of this work, we learned that Wallraff *et al.* have observed the Berry phase superconducting charge qubits with microwave techniques via quantum state tomography which are similar with the idea proposed in the paper [30].

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